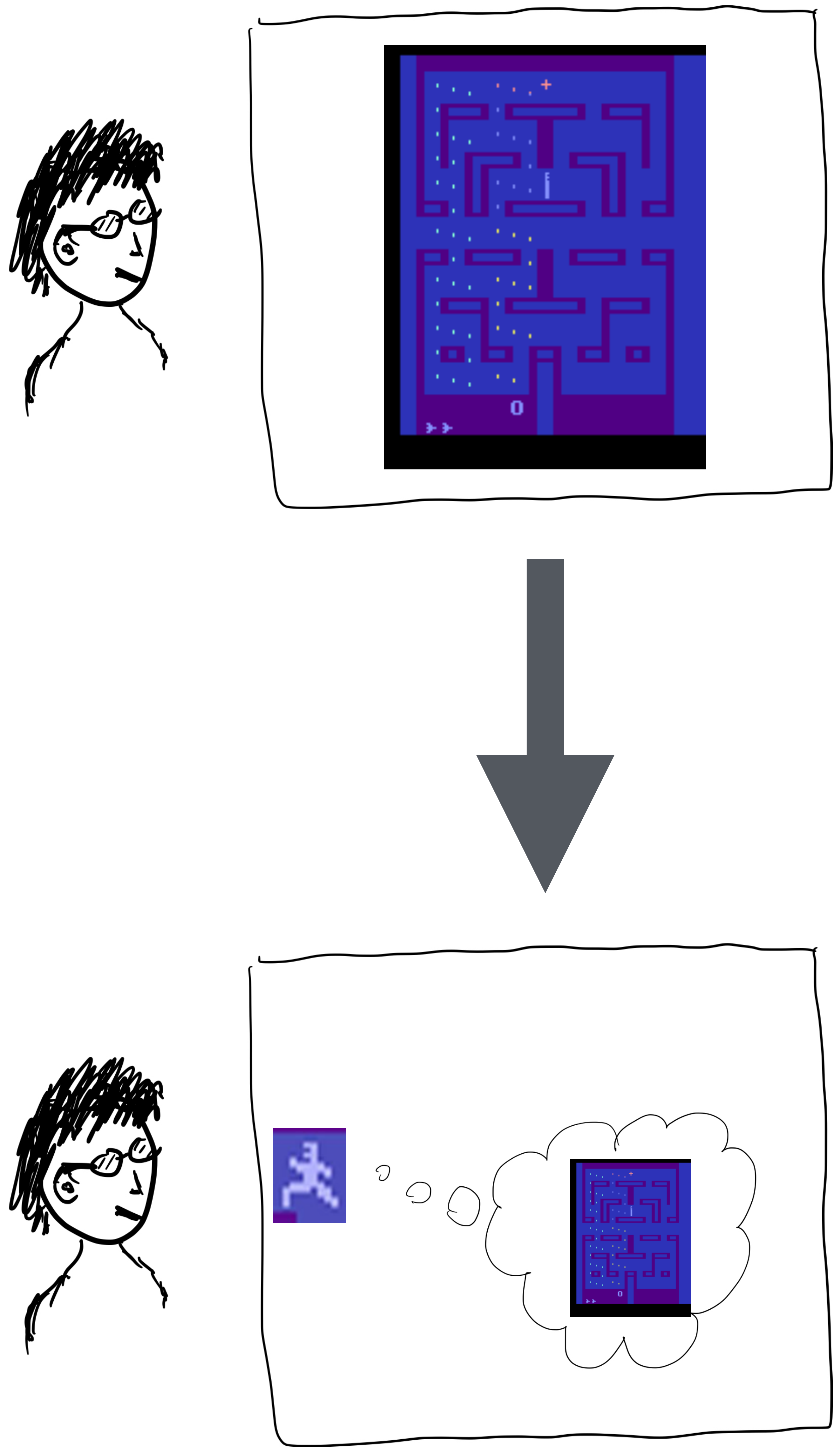
A Braitenberg vehicle's habitat

Manuel Baltieri - August 17th 2026, Artificial Life, Waterloo, Canada

ARAYA RESEARCH

Plan

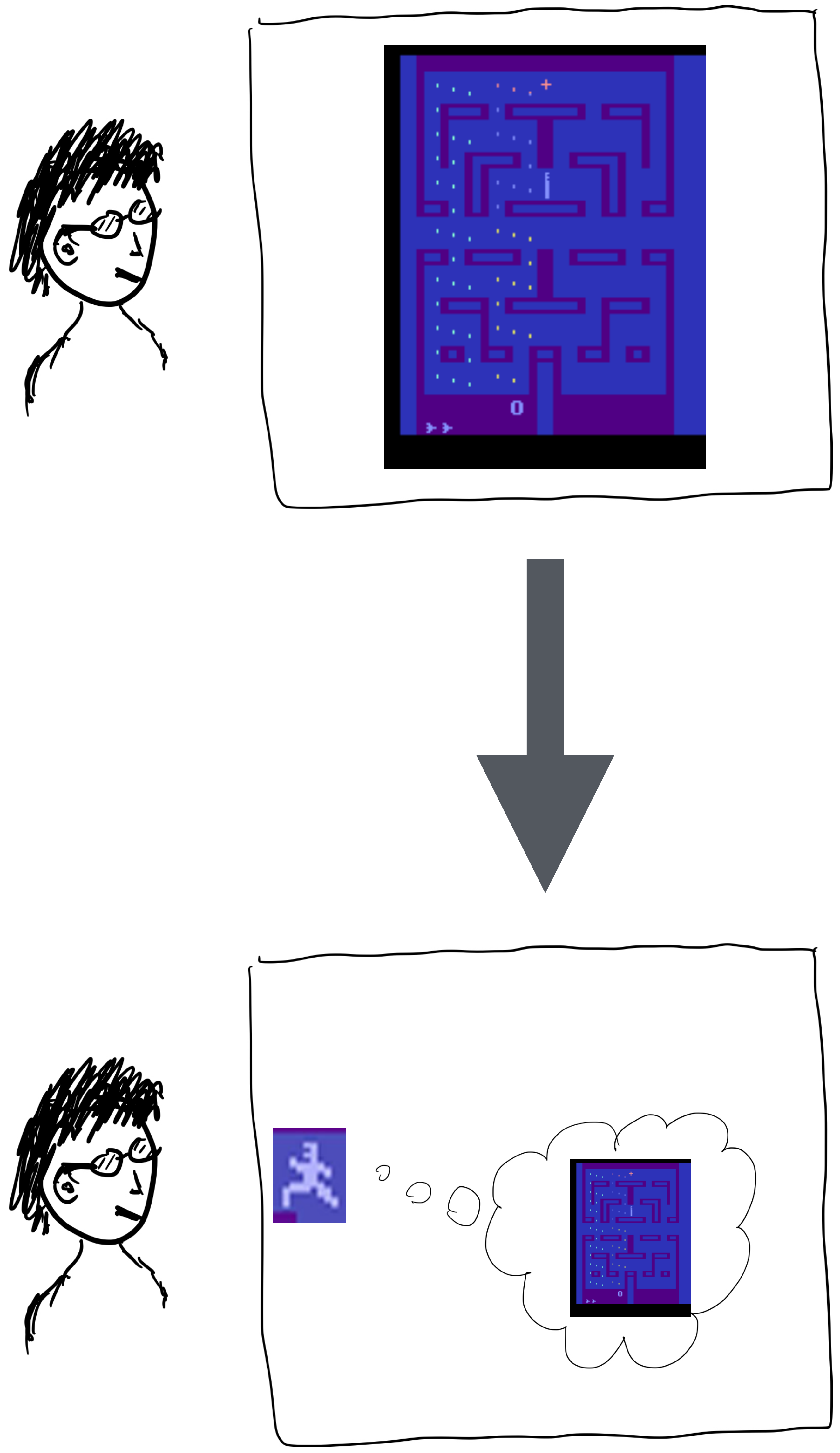
- Some background
- Environment vs habitat vs umwelt
- POMDPs (environment) —> quotient POMDPs (habitat) for Braitenberg vehicles
- Symmetries to obtain relevant quotients



Change of perspective

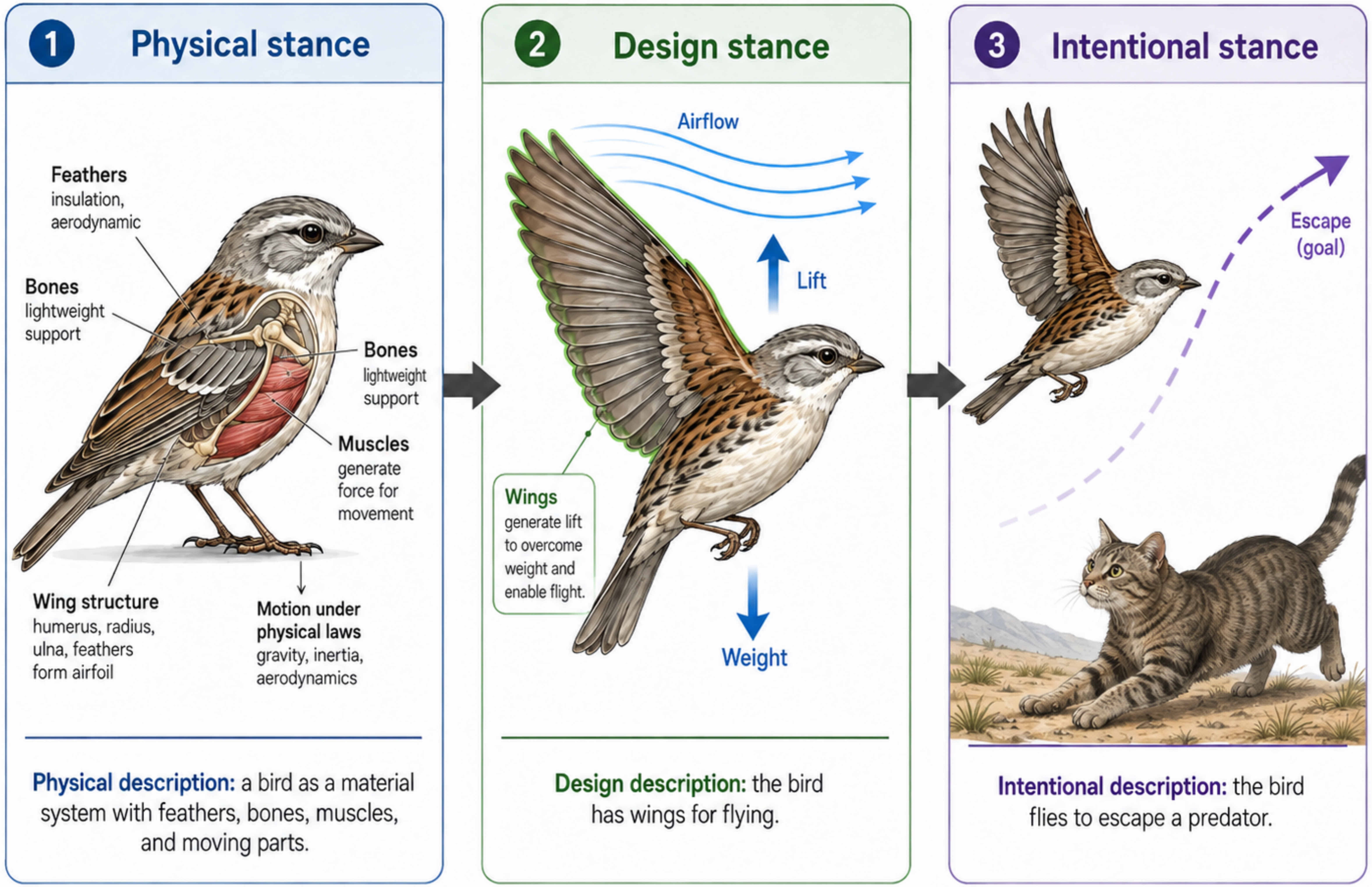
“What is it like to be a ~~bat~~ Braitenberg vehicle?”

“Maybe I just can’t know, because our experiences are (mostly) *invariant* to different things”



As-if agency

The intentional stance



Umwelt

World of or for an agent

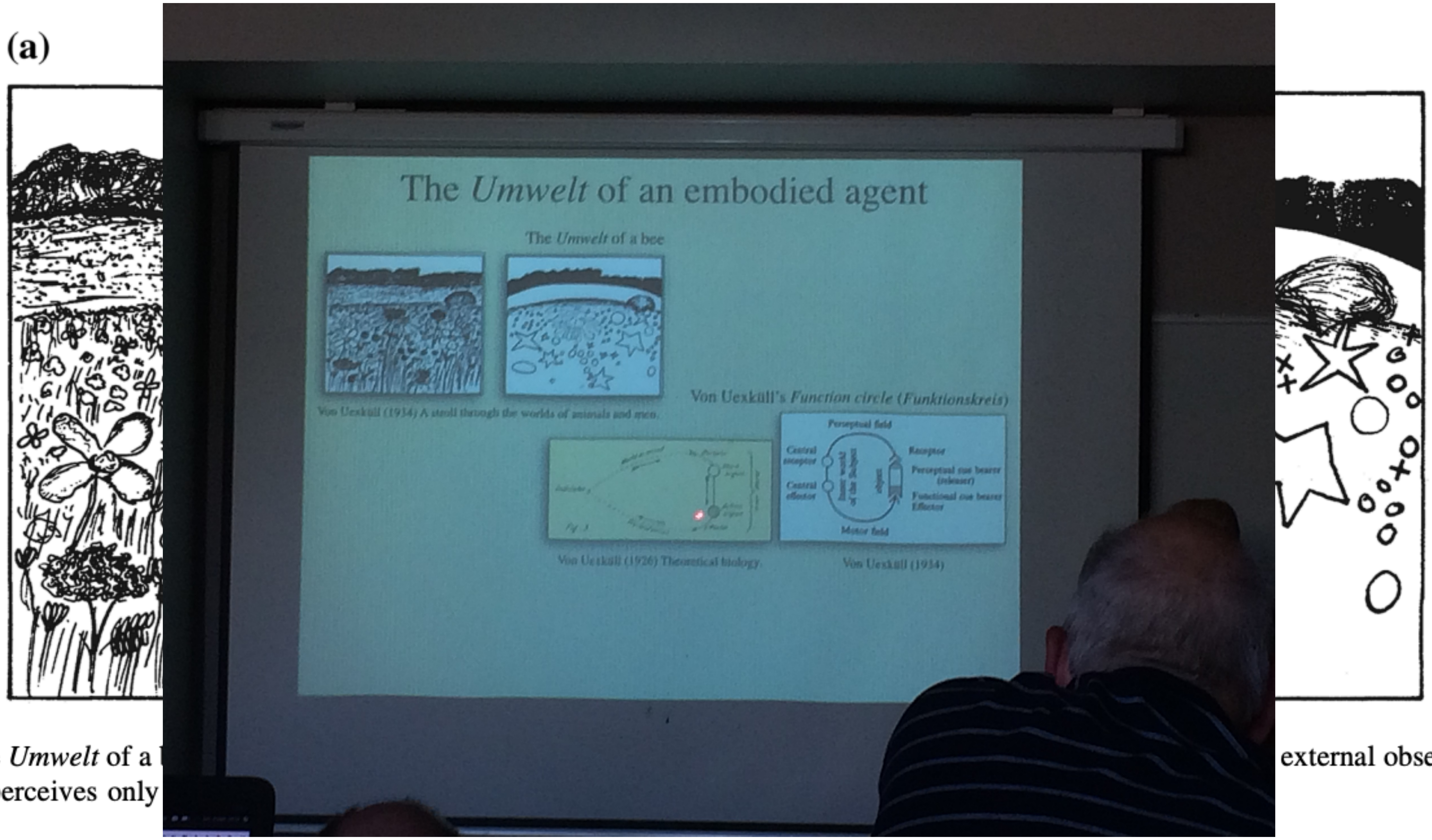
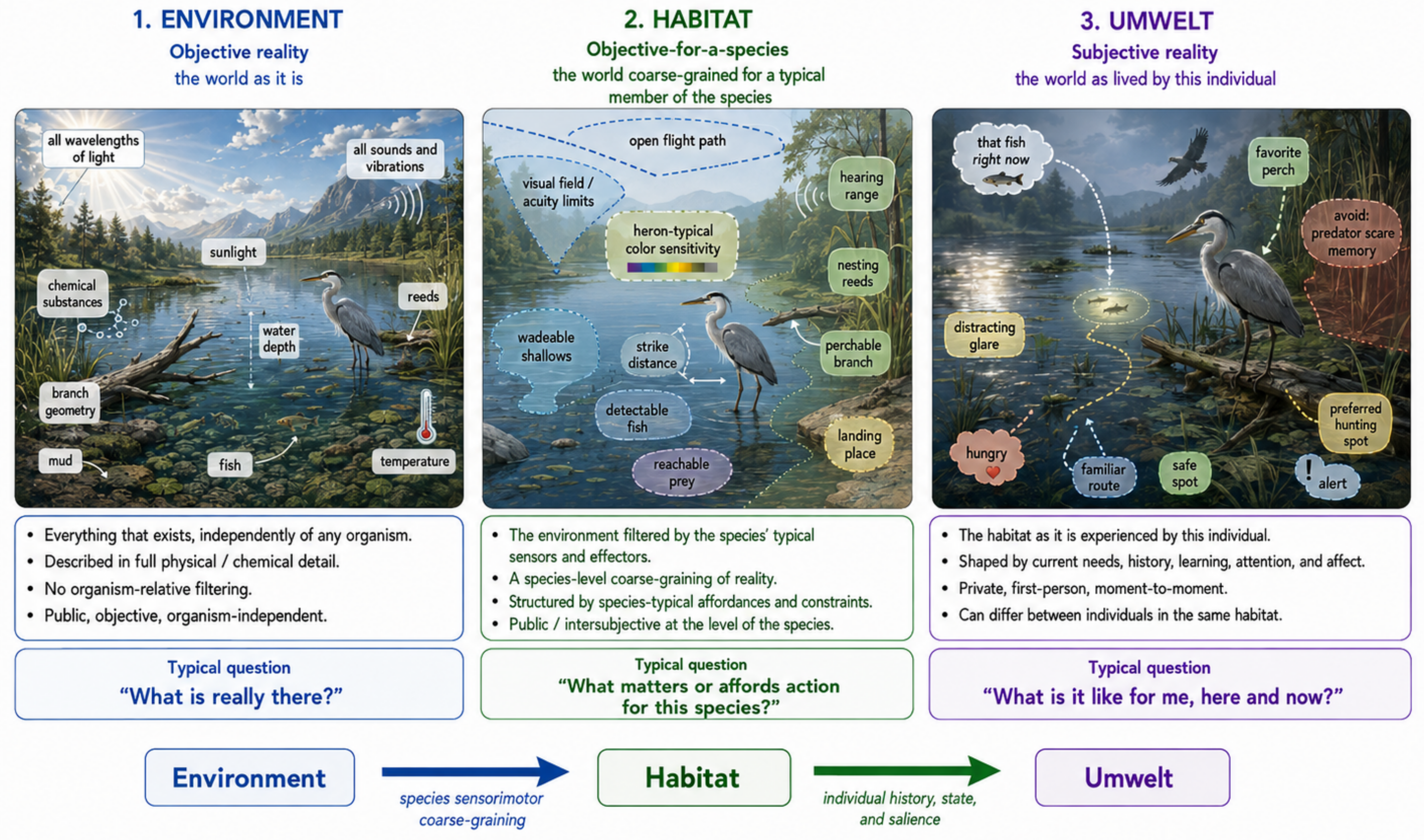


Fig. 1 The *Umwelt* of a … external observer. **b** The same bee perceives only …

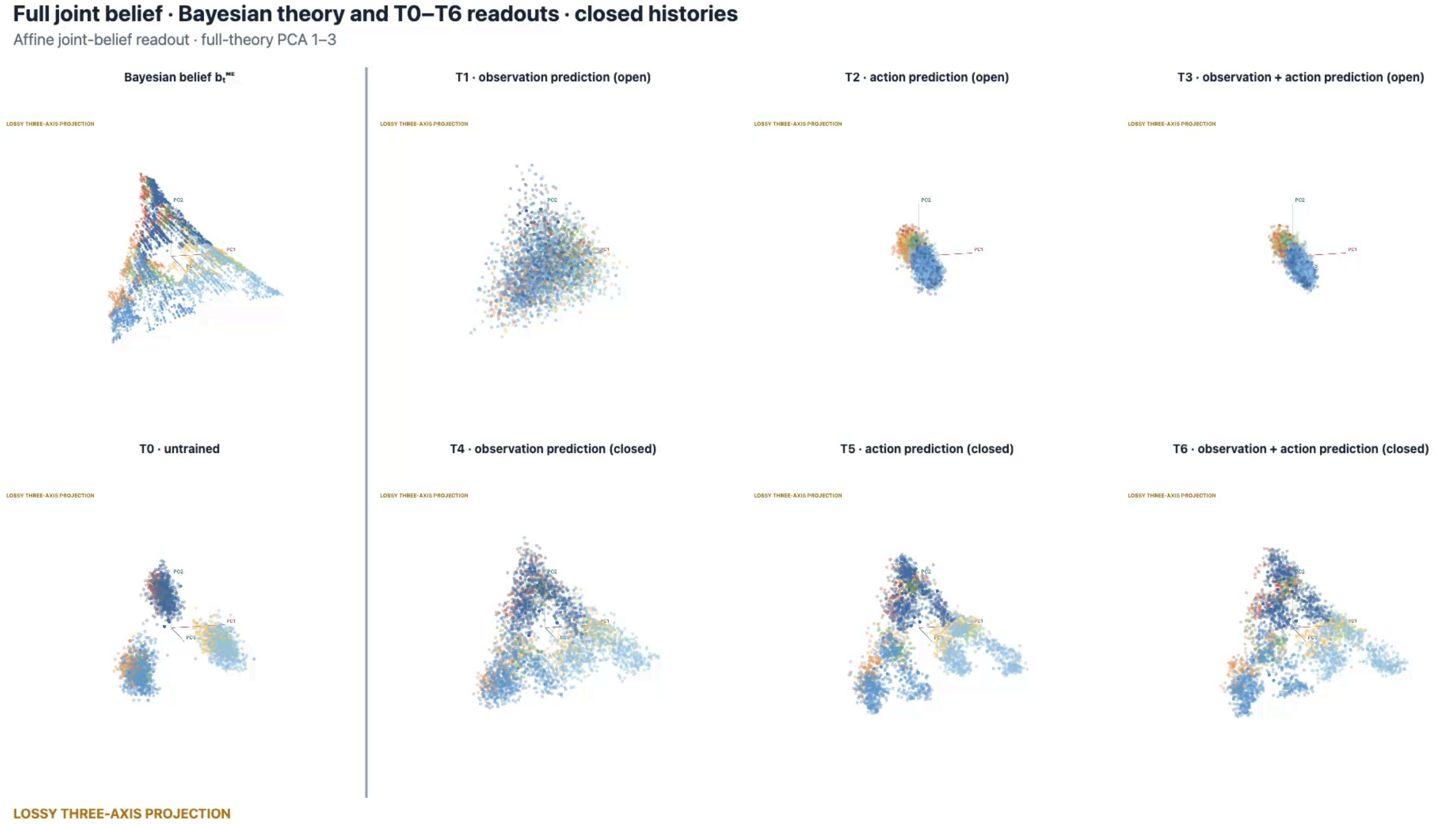
Habitat vs umwelt

Ay & Löhr (2015), Baggs & Chemero (2019, 2021) & ~~ Feiten (2020, 2024)



"But Braitenberg vehicles are too simple!"

Beliefs in transformers



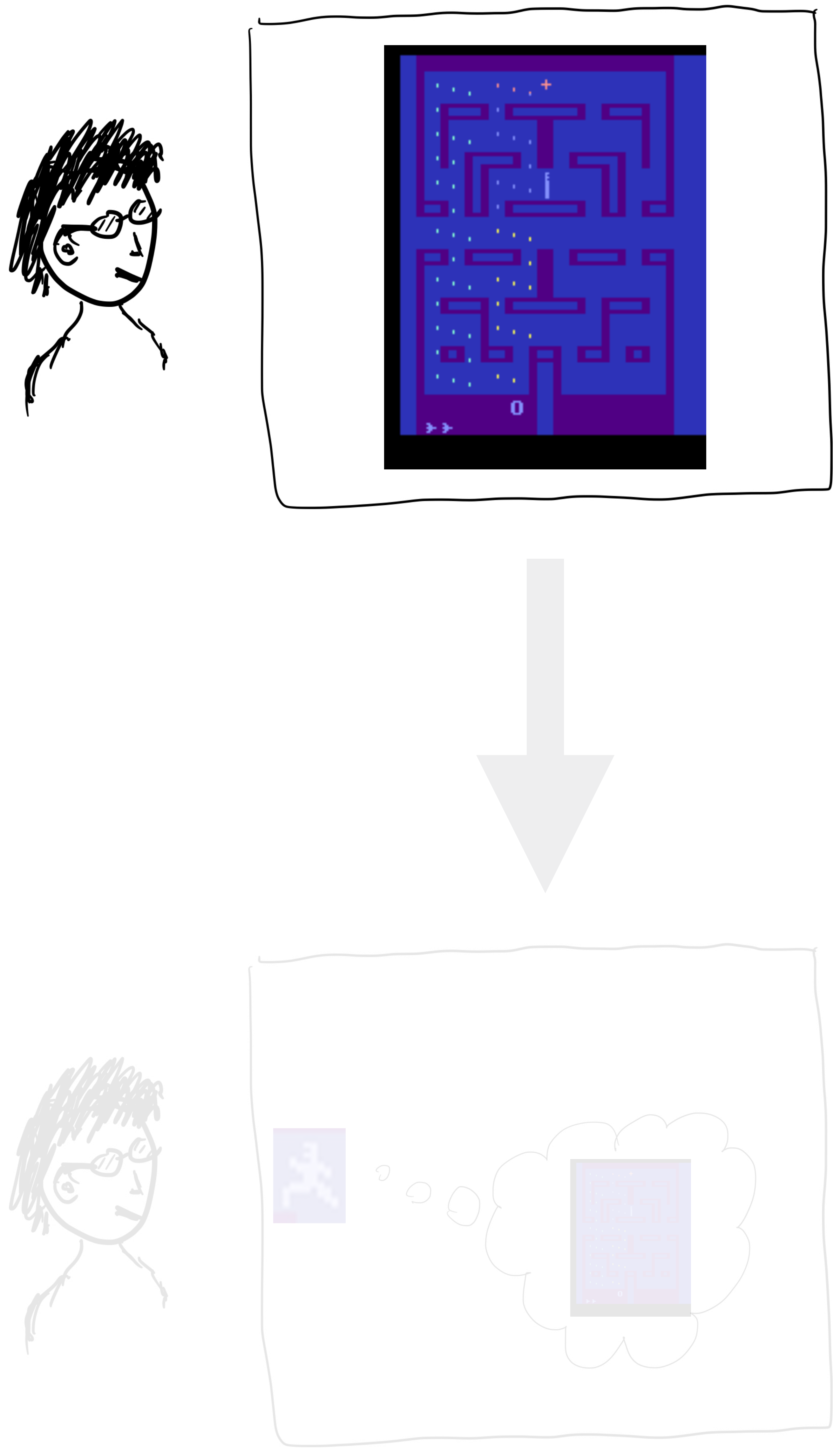
Environment —> Habitat

POMDPs to model environments

A POMDP (S, A, O, T, M) is given by

- State space, S
- Action space, A
- Observations, O
- Transition map, $T : S \times A \to P(S)$, often just written as $T(s' \mid s, a)$
- Observation map, $M : S \to P(O)$, often just written as $M(o \mid s, a)$

where $P(X)$ is the set of distributions on X.



Bisimulation equivalences of POMDPs

Given a POMDP, an equivalence relation $R \subseteq S \times S$ is a bisimulation equivalence if the following conditions hold for $C \in S/R$:

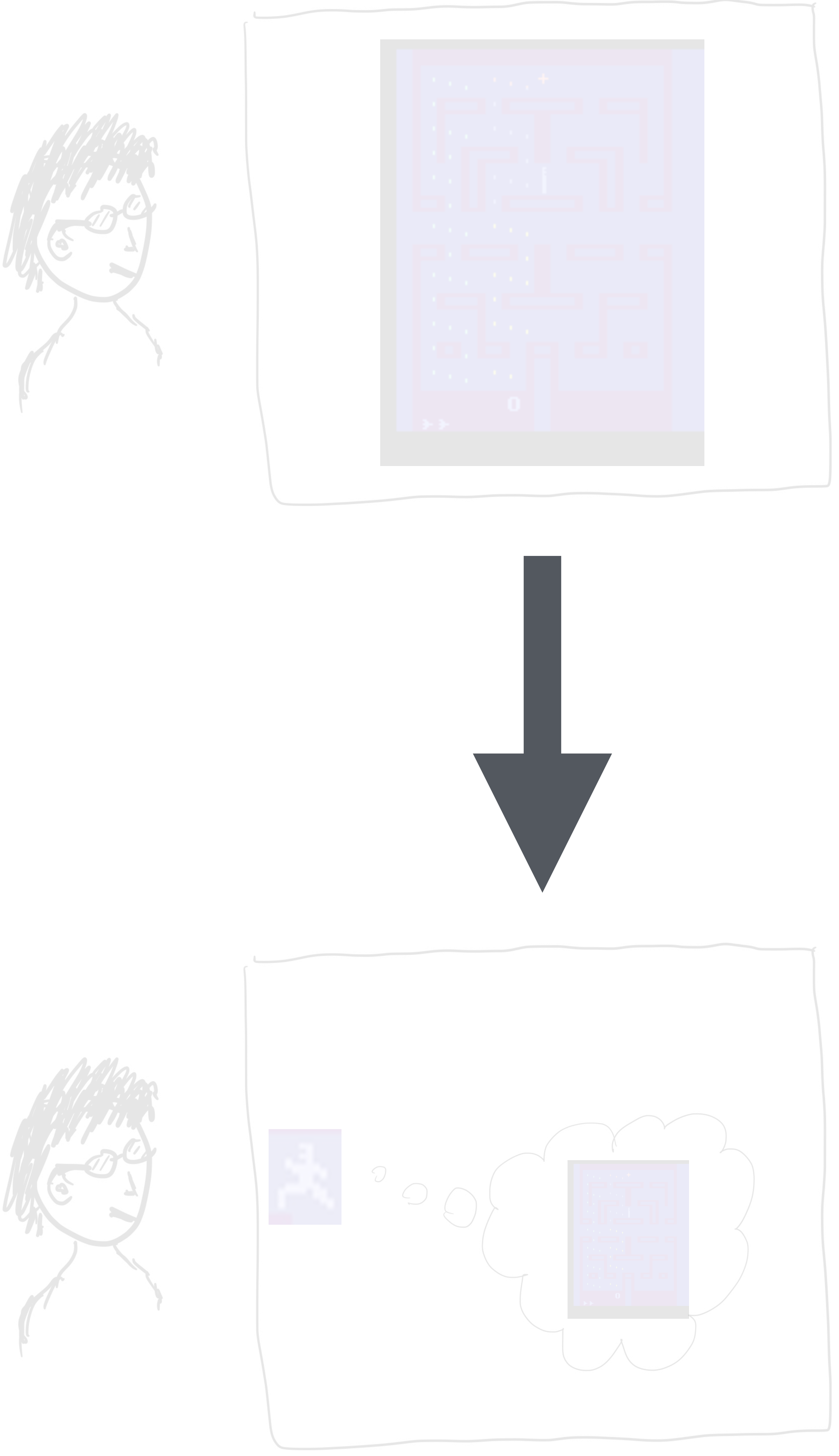
$$\sum_{s' \in C} T(s' \mid s, a) = \sum_{s' \in C} T(s' \mid \bar{s}, a)$$

(transitions)

$$M(o \mid s, a) = M(o \mid \bar{s}, a)$$

(observations)

and we say that $s \sim_R \bar{s}$ when $(s, \bar{s}) \in R$.



Quotient POMDP to model habitats

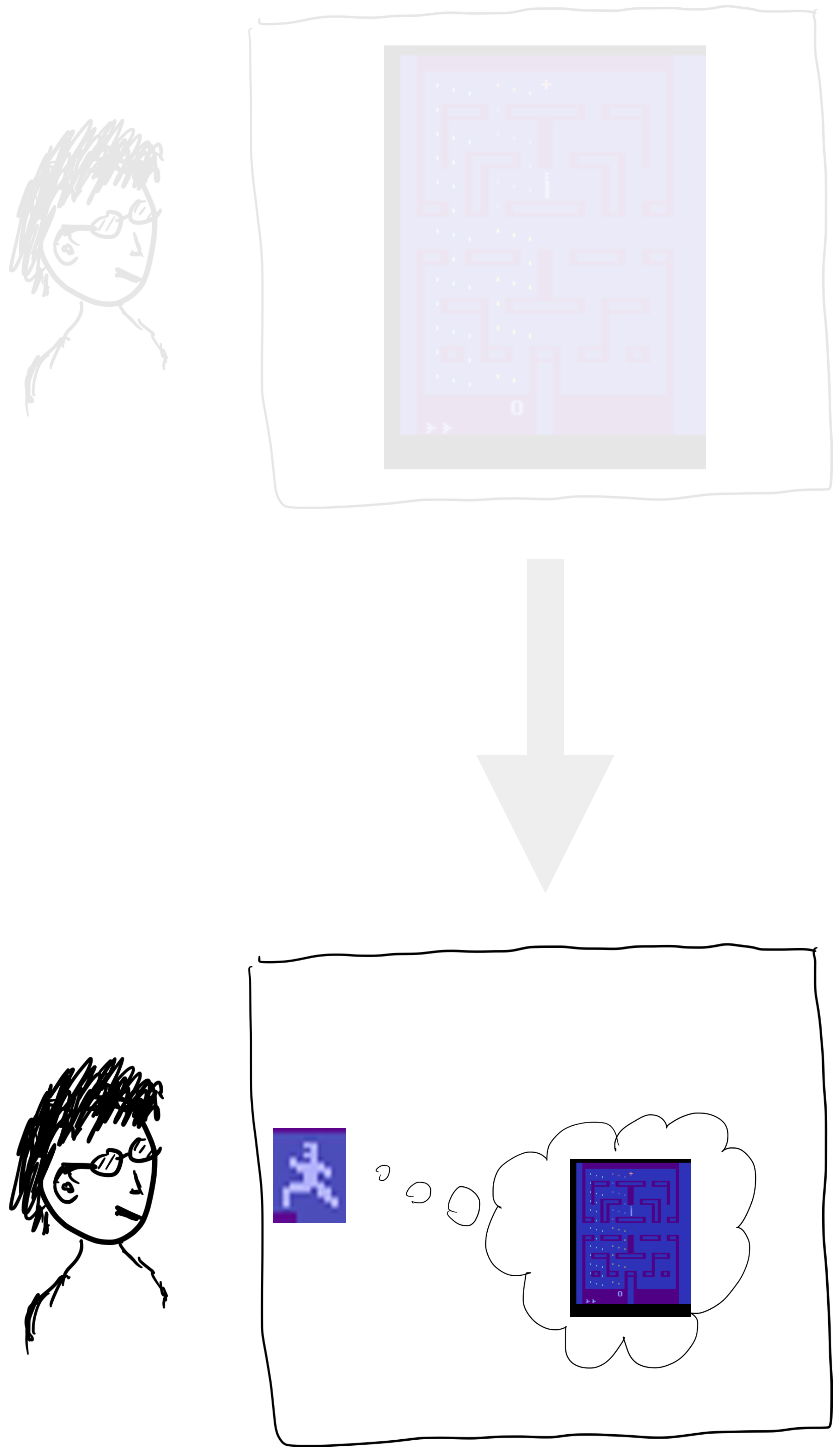
Given a POMDP $(S, A, O, T, M, G, \alpha_S, \alpha_A, \alpha_O)$ is the tuple $(S/R, A, O/R, \bar{T}, \bar{M})$, given by

- State space, orbit space S/R
- Action space, A
- Observation space: O/R
- Transition kernel $\bar{T} : (S/R) \times A \to P(S/R)$:

$$\bar{T}\left([s'] \mid [s], a\right) := \sum_{u \in [s']} T(u \mid s, a), \quad [s], [s'] \in S/R, s \in [s], a \in A$$

- Observation kernel $\bar{\Omega} : S/R_G \to P(O/R)$:

$$\bar{\Omega}([o] \mid [s]) := \sum_{v \in [o]} \Omega(v \mid s), \quad [s] \in S/R, [o] \in O/G, s \in [s]$$



Where does $\boldsymbol{R}$ come from?

For Braitenberg vehicles:

Property 1: **SO(2)** symmetry

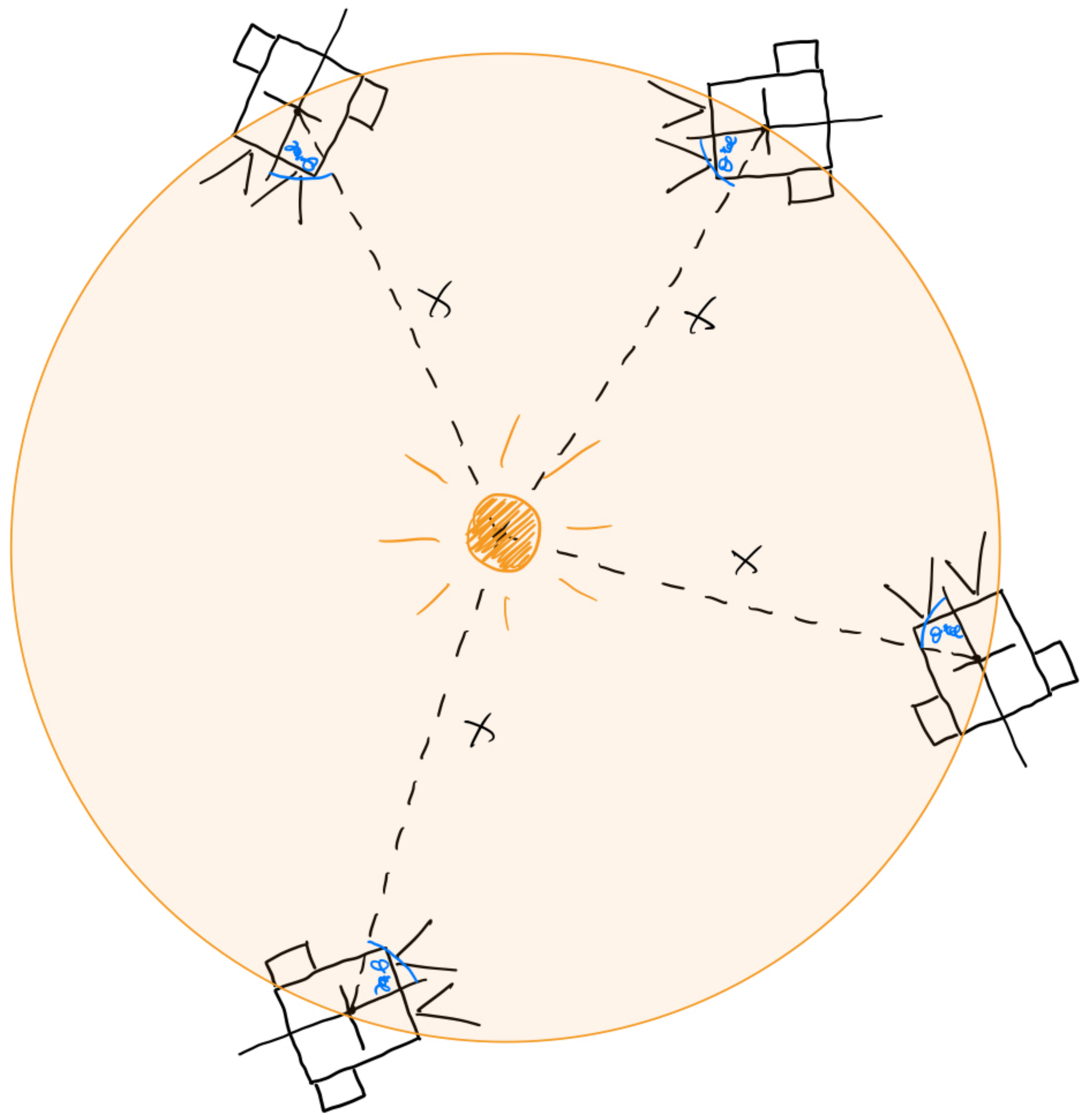
Invariance to rotations around source

Given

- same distance to source
- same attitude (angle θ^{rel})

there is an invariance to rotations centred around the source.

Rotating the world around the light, doesn't change sensations.

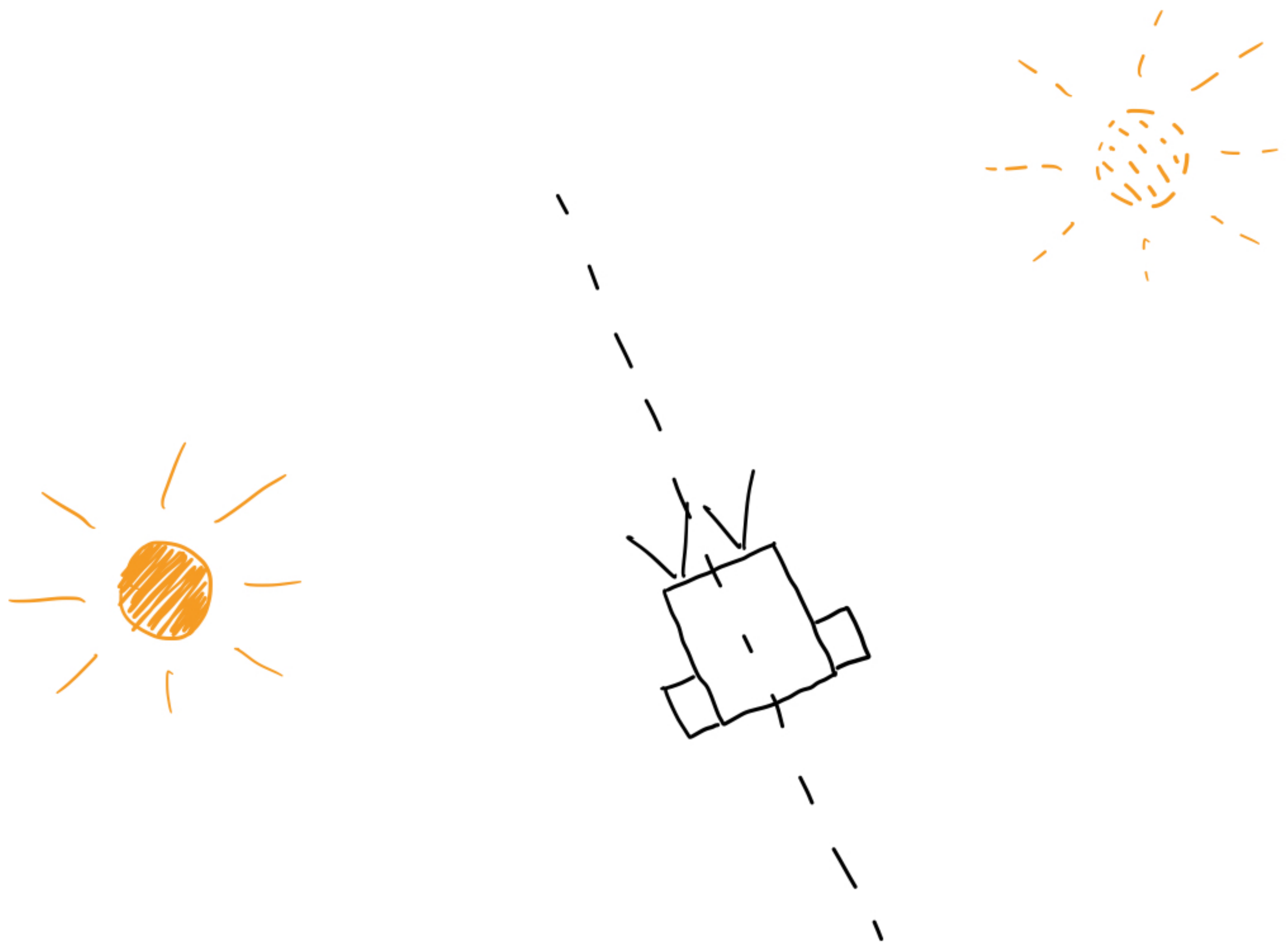


Property 2: $\mathbb{Z}_2$ symmetry

Equivariance to reflections along longitudinal axis

Given the longitudinal axis of the vehicle, we, there is an equivariance to reflections about this axis.

Left-right world reflections for the vehicle flip its sensory readings.

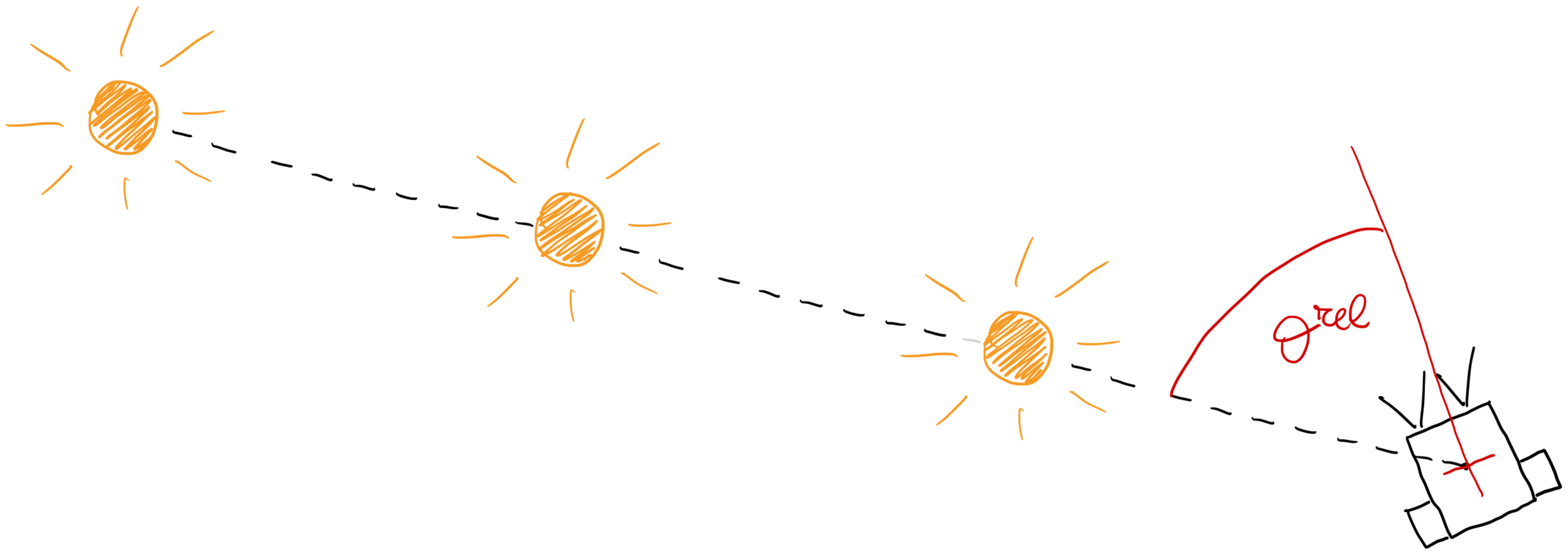


Property 3: $\mathbb{R}_{\geq 0}$ symmetry (?)

Equivariance to distance to light

Given the propagation law of the source and fixed bearing, sensory readings vary equivariantly with distance to the source.

Moving the vehicle along the line that connects it to the source, changes sensory readings by constants λ_1 and λ_2 fixed by propagation law.



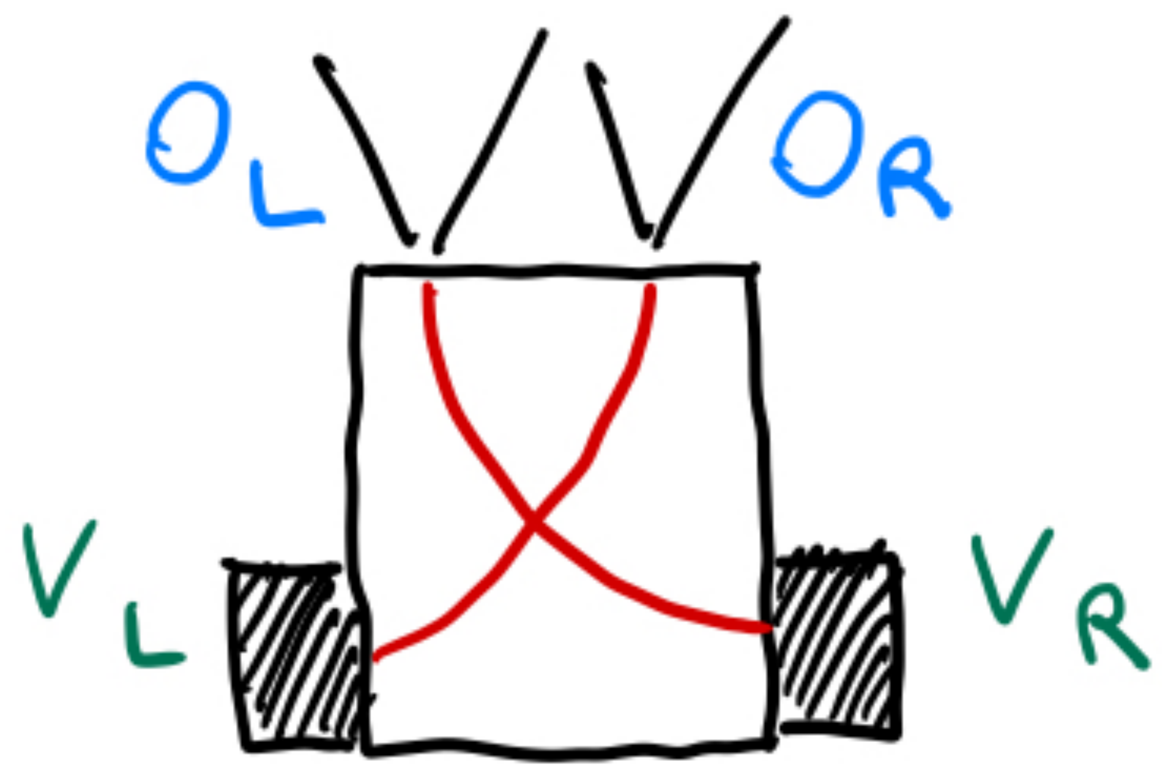
Agent as (stateless) policy

Trivial because of boundary choice

$\pi : O \to A$

$o_r \mapsto v_l = ko_r, \quad o_l \mapsto v_r = ko_l$

$$\frac{V_L}{O_R} = \text{constant} = \frac{V_R}{O_L}$$

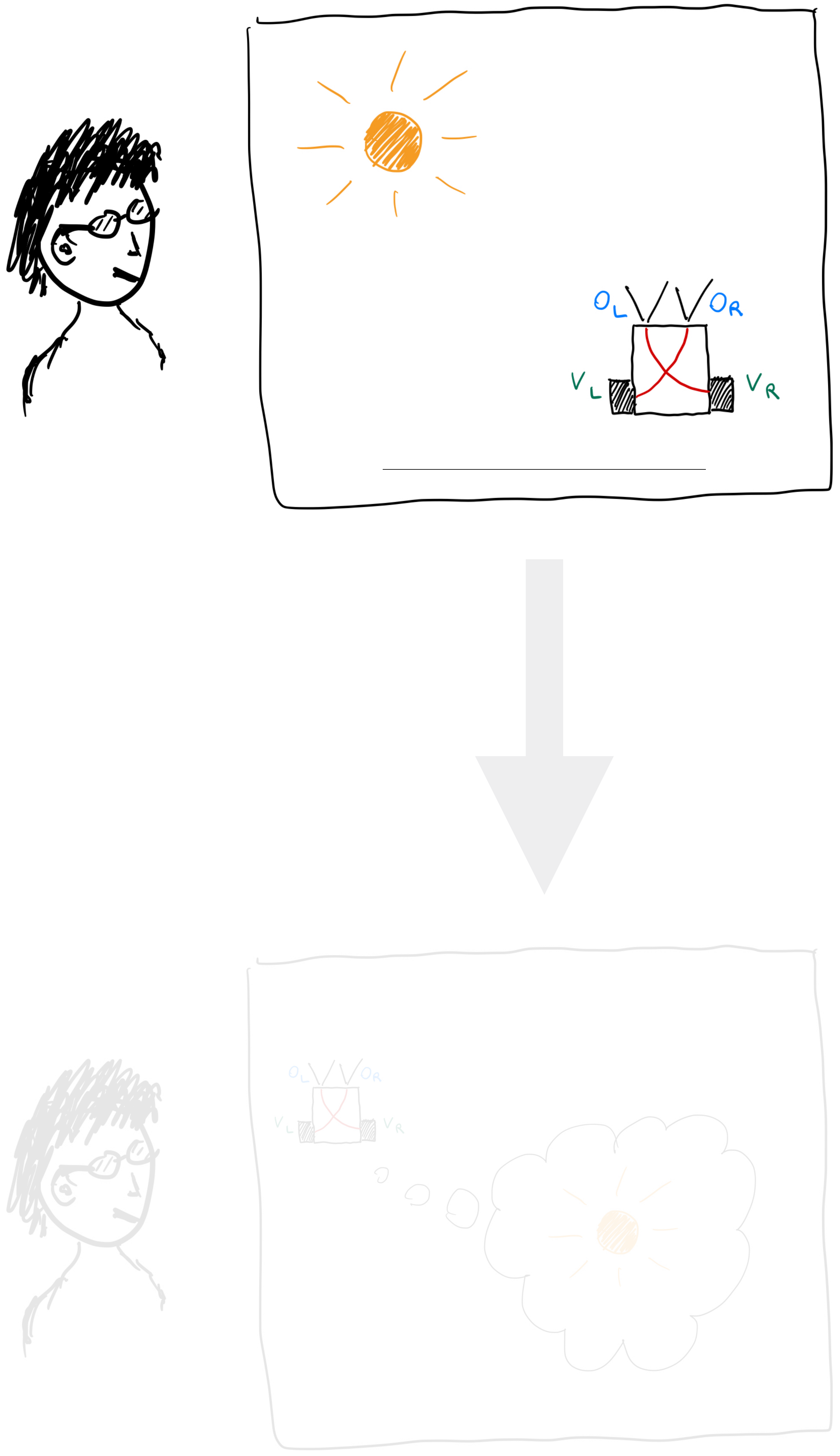


Symbol	Description
$X_{\text{Centre}} \subseteq \mathbb{R}^2$	Centre of vehicle's body
$\Theta \subseteq (-\pi, \pi]$	Absolute heading of the vehicle
$\Theta^{\text{rel}} \subseteq (-\pi, \pi]$	Heading relative to the stimulus
$X_{\text{Stim}} \subseteq \mathbb{R}^2$	Position of the stimulus
$R_{\text{Body}} \subseteq \mathbb{R}_{\geq 0}$	Radius of the vehicle body
$V_l \subseteq \mathbb{R}, V_r \subseteq \mathbb{R}$	Left/right linear velocities of wheels
$O_l \subseteq \mathbb{R}, O_r \subseteq \mathbb{R}$	Left/right sensory readings
$R \subseteq \mathbb{R}_{\geq 0}$	Radius of polar coord. (X_{Centre})
$\Phi \subseteq (0, 2\pi]$	Angle of polar coord. (X_{Centre})

Braitenberg POMDP

Physical environment

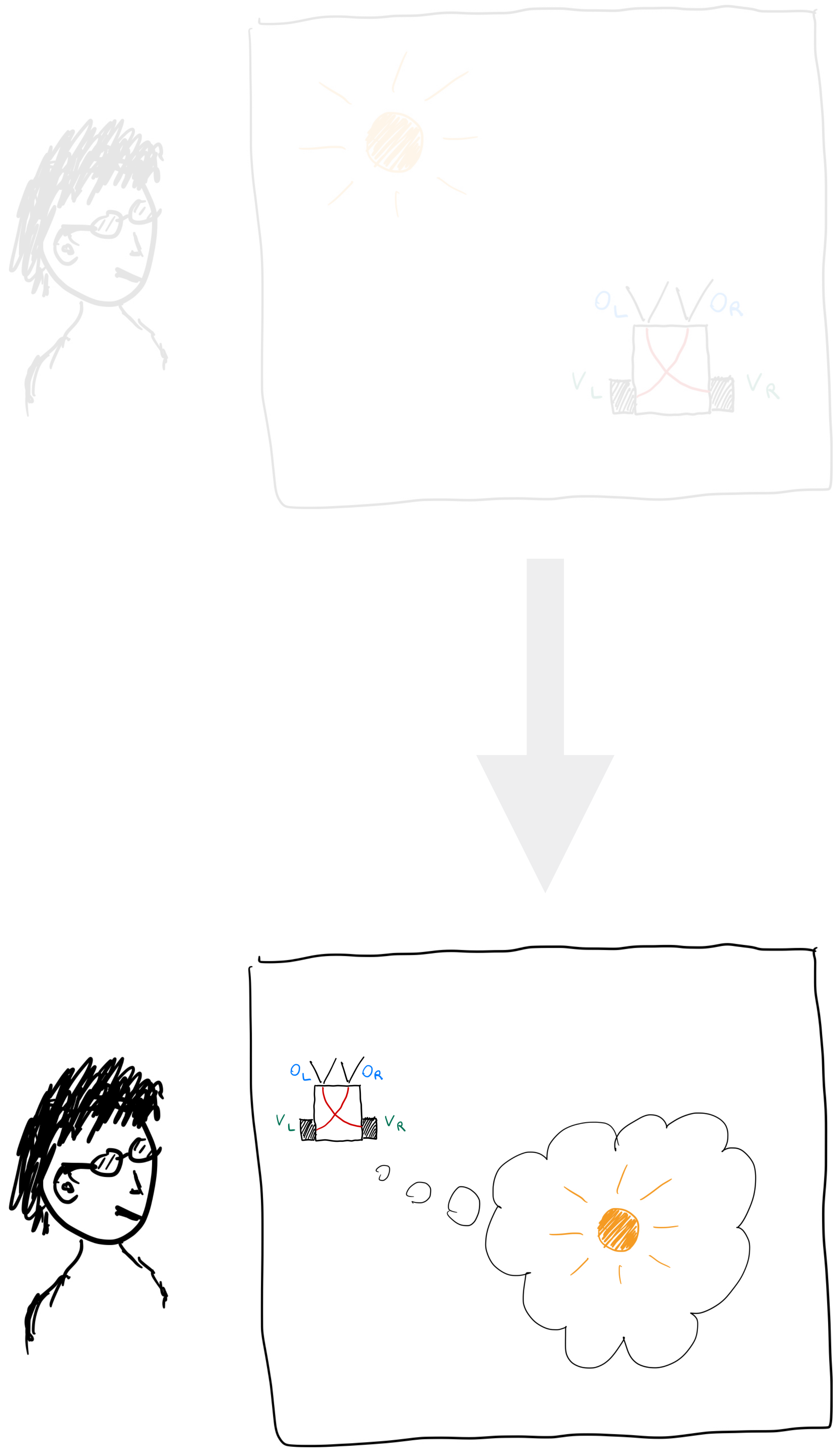
- State space, $S := X_{\text{Centre}} \times \Theta \times X_{\text{Stim}} \times R_{\text{Body}}$
- Action space, $A := V_l \times V_r$
- Observation space, $O := O_l \times O_r$
- Transition dynamics, position $f_{\text{Centre}} : X_{\text{Centre}} \times V_l \times V_r \to X_{\text{Centre}}$ and angle $f_\Theta : \Theta \times V_l \times V_r \to \Theta$
- Observation map, deterministic, given by the composition of f_{Sensor} and f_{Stim}, with $f_{\text{Sensor}} : X_{\text{Centre}} \times \Theta \times X_{\text{Stim}} \times R_{\text{Body}} \to X_l \times X_r \times X_{\text{Stim}}$, and $f_{\text{Stim}} : X_l \times X_r \times X_{\text{Stim}} \to O_l \times O_r$.



Braitenberg quotient POMDP

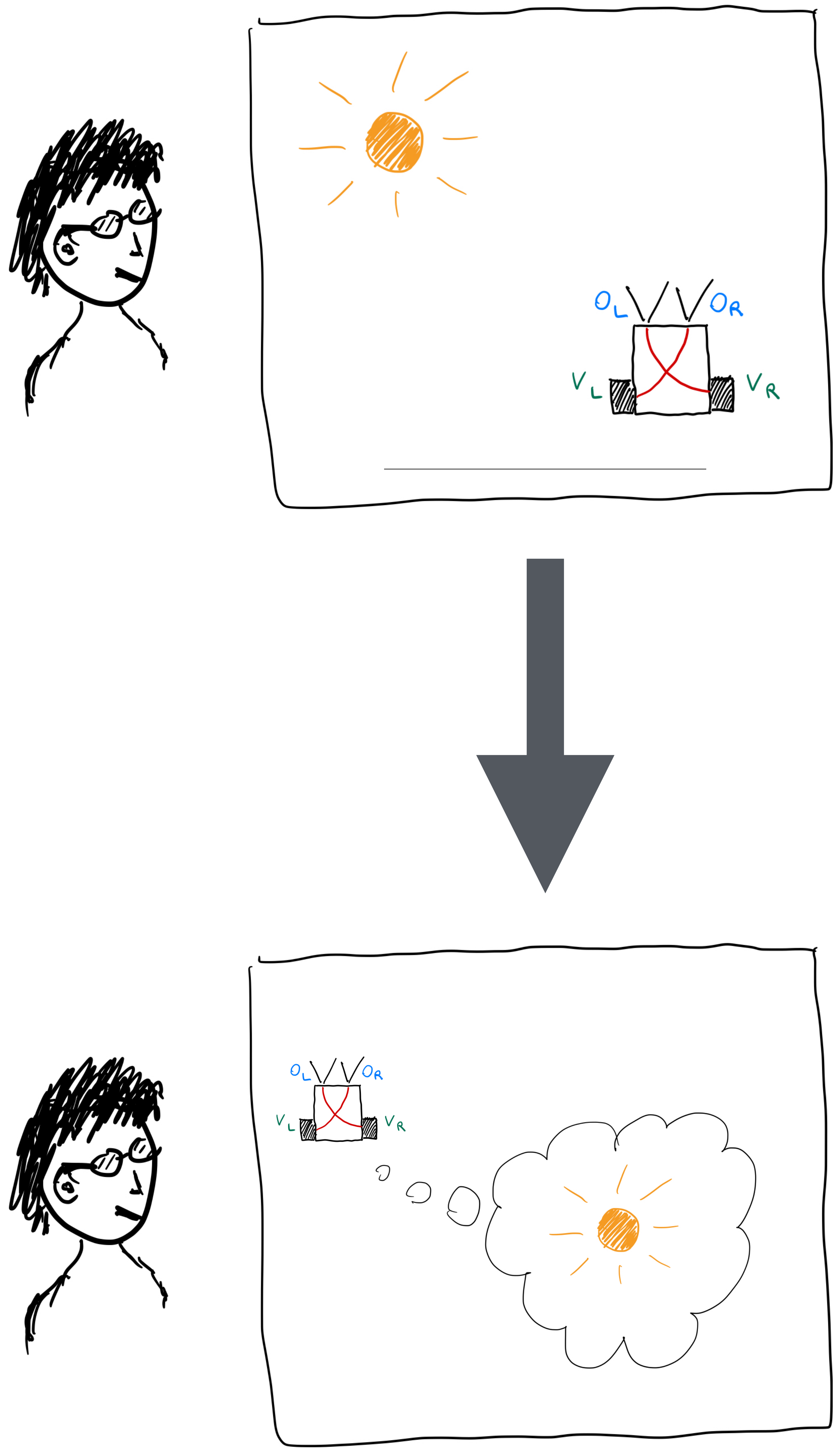
Use property 1

- State space, $S' := R \times \Theta^{\text{rel}}$
- Action space, $A := V_l \times V_r$
- Observation space, $O := O_l \times O_r$
- Transition dynamics, radius $f_R : R \times \Theta^{\text{rel}} \times V_l \times V_r \to R$ and relative angle $f_{\Theta^{\text{rel}}} : R \times \Theta^{\text{rel}} \times V_l \times V_r \to \Theta^{\text{rel}}$
- Observation map, deterministic, given by the composition of f_{Sensor} and f_{Stim}, with $f^{r\text{Body}}_{\text{Sensor}} : R \times \Theta^{\text{rel}} \to R_l \times R_r$, , and $f_{\text{Stim}} : R_l \times R_r \to O_l \times O_r$.



Take-home message

- Environment vs habitat vs umwelt
- Habitat ~= environment coarse graining capturing invariances for a class of agents
- Bisimulations can be somewhat arbitrary (but when induced by symmetry groups of interest less so)
- Next: disentangle full group structure + study umwelt + quotienting environment vs. agent



Acknowledgements

- Coauthors: Fernando Rosas and Filippo Torresan
- For discussions about this: Martin Biehl, Artemy Kolchisky, Nathaniel Virgo, Felix Woolford, Anil Seth, Warrick Roseboom, and Peter Lush
- Funders: Moonshot R&D, Grant Number JPMJMS2012
- Audience: you!